| Name: Answer k | Sey |
|----------------|------------|
| Mr. Denton | |

GRAPIER TEST

Part 1. $(50\,\mathrm{points})$ Please attempt all of the the following true or false and multiple choice problems. Partial credit is not available for Part 1.

| $(2_{ea.}^{pts})$ | 1. True or False. Determine if the statement is | true or false. | |
|------------------------|--|---|-------|
| (cu.) | (a) $\underline{\hspace{1cm}}$ The statement $a < b$ is read "a is less than b". | | |
| | (b) $\overline{\mathbf{T}} a - (-b) = a + b.$ | | |
| | (c) F The number 1 is a prime number because its only factors are 1 and itself. | | |
| | (d) We can add and subtract fractions only if they have the same denominator. | | |
| | (e) Every percent can be rewritten as a fraction with a denominator of 100. | | |
| | (f) T Multiplying a decimal number by 10 moves the decimal point one place to the right. | | |
| | (g) \underline{F} Dividing a decimal number by 10 moves the decimal point one place to the right. | | |
| | (h) F Performing any multiplications comes before simplifying any exponents when using the order of operations. | | |
| | (i) Factoring and using the distributive | ve property have opposite mathematical effects. | |
| $\left(4^{pts}\right)$ | 2. The absolute value $ a $ of a real number a call be any positive real number. \Box be equal to zero. | nn NOT be a negative real number. all of the above. | 4 pts |
| (4 ^{pts}) | 3. Express as a mixed number: $\frac{53}{8}$ | $\begin{array}{ c c } \hline \checkmark & 6\frac{5}{8} \\ \hline \hline & 4\frac{3}{8} \\ \hline \end{array}$ | 4 pts |
| (4 ^{pts}) | 4. Express as an improper fraction: $7\frac{2}{9}$ | $\begin{array}{ c c }\hline & \frac{27}{9}\\\hline & \frac{18}{9}\end{array}$ | 4 pts |

| Elementary Algebra/Test.Chapter.01 - Page 2 of 6 - Name: Answer K | Elementary Algebr | a/Test.Chapter.01 | – Page 2 of 6 – | Name: | Answer Ke |
|---|-------------------|-------------------|-----------------|-------|-----------|
|---|-------------------|-------------------|-----------------|-------|-----------|

(4^{pts})**5.** What is the least common multiple (LCM) of the numbers 2, 4, and 7?

 $4 \, \mathsf{pts}$

 \square 7

(4^{pts})

6. Which of the following represents the exponential expression 2^34^5 ?

 $2 \cdot 3 \cdot 4 \cdot 5$

2+3+4+5

2+2+2+4+4+4+4+4

 $4 \, \mathrm{pts}$

7. Evaluate $x^2 - 5x - 32$ for x = -8(4^{pts})

72

136

 $4 \, \mathsf{pts}$

(4^{pts})**8.** The property a(b+c)=ab+bc, which holds for all real numbers a, b, and c, is known as

 $4\,\mathrm{pts}$

Associative property.

Distributive property.

Commutative property.

Zero-factor property.

(4^{pts})**9.** Simplify: 5x - 9 - 2x - 18.

 $4\,\mathrm{pts}$

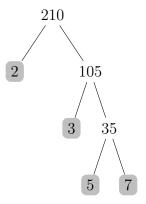
3x-9

Part 2. (50 points) Please attempt all of the following open-ended problems. You must show all work to receive full credit. Partial credit is available for Part 2.

(4^{pts})**1.** Use a factor tree to write the prime factorization of 210.

 $4 \, \mathrm{pts}$

Solution:



We therefore conclude that $210 = 2 \cdot 3 \cdot 5 \cdot 7$.

(4^{pts}) **2.** Simplify $\frac{2}{9} \div \frac{8}{21}$ to lowest terms.

Solution:

4 pts

To start, we change the division problem to a multiplication problem by multiplying by the reciprocal of the dividend. That give us the equivalent problem $\frac{2}{9} \cdot \frac{21}{8}$. Next we factor all the denominators and numerators, canceling any factors common to both. That gives us now that

$$\frac{2}{9} \div \frac{8}{21} = \frac{2}{9} \cdot \frac{21}{8} = \left(\frac{2}{3 \cdot 3}\right) \left(\frac{3 \cdot 7}{2 \cdot 2 \cdot 2}\right) = \frac{2 \cdot 3 \cdot 7}{3 \cdot 3 \cdot 2 \cdot 2 \cdot 2} = \frac{7}{3 \cdot 2 \cdot 2} = \frac{7}{12}$$

(4pts) **3.** Rewrite 0.42 as a fraction. Your answer should be in lowest terms. Show all of your work! *Solution*:

4 pts

We note that

$$0.42 = \frac{42}{100} = \frac{2 \cdot 3 \cdot 7}{2 \cdot 2 \cdot 5 \cdot 5} = \frac{3 \cdot 7}{2 \cdot 5 \cdot 5} = \frac{21}{50}$$

(8^{pts}) **4.** Simplify the expression $\frac{4^2+3^2}{3+4\cdot 13}$. Show all of your work!

8 pts

Solution: We begin by simplifying our numerator and denominator. For the numerator, we have that $4^2+3^2=16+9=25$, and for the denominator, we have that $3+4\cdot 13=3+52=55$. It now follows that

$$\frac{4^2+3^2}{3+4\cdot 13} = \frac{25}{55} = \frac{5\cdot 5}{5\cdot 11} = \frac{5}{11}$$

(6^{pts}) **5.** The following data set represents the number of touchdown passes completed by Joe Montana in each of his 16 seasons in the NFL: $\{1\ 15\ 19\ 17\ 26\ 28\ 27\ 8\ 31\ 18\ 26\ 26\ 0\ 2\ 13\ 16\}$.

6 pts

(a) (2 pts) Find the mean of this data set.

Solution:

$$\begin{array}{ll} \mathsf{mean} & = & \frac{1+15+19+17+26+28+27+8+31+18+26+26+0+2+13+16}{16} \\ & = & \frac{273}{16} = 17.0625 & \leadsto & \boxed{\mathsf{mean} = 17.0625} \end{array}$$

(b) (2 pts) Find the median of this data set.

Solution: Reordering as a strictly increasing sequence of integers we find

$$0\ 1\ 2\ 8\ 13\ 15\ 16\ \boxed{17\ 18}\ 19\ 26\ 26\ 26\ 27\ 28\ 31$$

from which it follows that the $\frac{17+18}{2}=17.5$

(c) (2 pts) Find the mode of this data set. *Solution*:

By Inspection, only one number occurs more than once, and therefore the $\boxed{\text{mode} = 26}$

(6^{pts}) **6.** Simplify the expression 7y - 8(2y - 30). Show all of your work! *Solution*:

$$7y - 8(2y - 30) = 7y + (-8)(2y - 30)$$

$$= 7y + (-8)(2y) - (-8)(30)$$

$$= 7y + (-16)y - (-240)$$

$$= 7y - 16y + 240$$

$$= -9y + 240$$

(6^{pts}) **7.** Evaluate the expression $x^2 + 6x - 17$ for x = -8. Show all of your work! *Solution*:

6 pts

$$(-8)^{2} + 6(-8) - 17 = 64 + 6(-8) - 17$$
$$= 64 - 48 - 17$$
$$= -1$$

(8^{pts}) **8.** Evaluate the expression $b^2 - 4ac$ for a = 2, b = 4 and c = 2. Show all of your work! *Solution*:

8 pts

$$(4)^{2} - 4(2)(2) = 16 - 4(2)(2)$$

$$= 16 - 16$$

$$= 0$$

- (4^{pts}) **9.** Build a variable expression that represents the following phrases.
 - (a) (1 pt) "Seven less than four times a number".

 $4\,\mathrm{pts}$

Solution:

Let x be the number. Then the expression is 4x - 7.

(b) (1 pt) "Seven more than one half of a number".

Solution:

Let x be the number. Then the expression is $\frac{1}{2}x + 7$.

(c) (1 pt) "The product of four with three different numbers".

Solution:

Let x, y, and z be the numbers. Then the expression is 4xyz.

(d) (1 pt) "The quotient of a number and π ".

Solution:

Let x be the number. Then the expression is $\frac{x}{\pi}$.



Part 3. (20 bonus points) You must first attempt all of the questions from Part 1 to receive any credit for solutions to the \clubsuit and \diamondsuit questions. Furthermore, you must also first attempt all of the questions from Part 2 to receive any credit for solutions to the \heartsuit and \spadesuit questions. Partial credit is available for Part 3.

(2 bonus points) A group of four friends went out to dinner. If each person paid \$23, what was the total bill?

Solution:



 \diamondsuit (4 bonus points) Jim Rockford, a private investigator from the 1970's TV show *The* Rockford Files, charged \$200 per day plus expenses to take a case. Letting d represent the number of days Rockford worked on a case and assuming that he had \$425 in expenses, build a variable expression for the amount of money he would charge for the case.

Solution:

Since Rockford worked d days, he is owed \$200d. His expenses cost him an additional \$425. Therefore the variable expression for the amount of money he would charge for the case is

$$$200d + $425$$



 \heartsuit (6 bonus points) If one recipe calls for $1\frac{1}{2}$ cups of flour and a second recipe calls for $2\frac{2}{3}$ cups of flour, how much flower is needed to make both recipes?

Solution:

To add mixed numbers we must first convert them to improper fractions. To that end we note that $1\frac{1}{2}=3/2$ and $2\frac{2}{3}=8/3$. To add fractions with different denominators, we need to use the least common multiple to find a common denominator. The LCM of 2 and 3 is 6, and so we write $\frac{3}{2} = \left(\frac{3}{3}\right)\frac{3}{2} = \frac{9}{6}$ and $\frac{8}{3} = \left(\frac{2}{2}\right)\frac{8}{3} = \frac{16}{6}$. We can now add, and we have that

$$1\frac{1}{2} + 2\frac{2}{3} = \frac{9}{6} + \frac{16}{6} = \frac{25}{6} = 4\frac{1}{6}$$

We therefore conclude that $4\frac{1}{6}$ cups of flour are needed to make both recipes.



(8 bonus points) The following data set represents the number of touchdown passes completed by Joe Montana in each of his 16 seasons in the NFL: {1 15 19 17 26 28 27 8 31 18 26 26 0 2 13 16. For two points each calculate both the range and mid-range, and then complete a bar graph that shows the number of touchdowns completed for seasons five through nine.

Solution:

mid-range= $\frac{31+0}{2} = \frac{31}{2} = \boxed{15.5}$ range=(31)-(0)=3135 30 25 20 15 10 5

0

5

6

7

8

9

Lesson Plan

- Instructor: Luke Denton
- Subject: Elementary Algebra
- Textbook: G. Woodbury, Elementary and Intermediate Algebra, Third edition
- Section: 1.1-Integers, Opposites and Absolute Values
- Objectives: Students will be able to
 - ► Graph whole numbers on a number line
 - ▶ Determine which is the greater of two whole numbers
 - ► Graph integers on a number line
 - ► Find the opposite of an integer
 - ▶ Determine which is the greater of two integers
 - ► Find the absolute value of an integer
- Classroom Instructional Materials:
 - ▶ **Teacher:** Whiteboard and Pen; Computer and Projector; Lesson Plan.
 - ▶ Students: Pencil and Paper; Structured Notes (optional).
- Accommodations and Modifications: As documented/needed.
- Total Time Allotted: 55 minutes
 - Opening (5 minutes)
 - S Focus Lesson (20 minutes)
 - Checking for Understanding and Lesson Closure (5 minutes)
 - **Guided Practice (10 minutes)**
 - (15 minutes)
- Homework Assignment: 30-45 minutes
 - Pages 5-6: 5,7,11,13,17,23,25,27,31,35,37,39,43,45,47,49,51,59,61

Opening

- The Art of Math Picture of the Day
- · Class Announcements
- Introduce Lesson Objectives

Focus Lesson

• **Definition:** A **Set** is a collection of elements or members. A set is indicated by placing the elements of the set between two braces.

- **Example:** The set containing the first five prime numbers is denoted by {2,3,5,7,11}.
- Definition: A Subset of a set is a collection of some or all of its elements or members.
- Example: In the set of all animals, the set of all mammals is a subset.
- Definition: A set with no elements is called the Empty Set. It is represented by the symbol θ.
- Example: The set of all animals that are both real and mythical is an empty set.
- Definition: A Rational Number is any number that can be written as a fraction.
- **Example:** The fraction 4/7 is a rational number.
- **Example:** Any decimal number that terminates, for example the decimal number 0.247, is a rational number.
- **Example:** Any decimal number that repeats, for example the decimal number $0.\overline{333}$, is a rational number.
- **Definition:** An *Irrational Number* is any number that can not be written as a fraction, and instead is a decimal number that does not repeat or terminate.
- **Example:** The most famous irrational number is probably the number $\pi \approx 3.14159...$
- **Definition:** The set of *Natural Numbers* is the set $\{1, 2, 3, ...\}$.
 - The natural numbers are also known as "the counting numbers".
- **Definition:** The set of *Whole Numbers* is the set $\{0, 1, 2, 3, ...\}$.
 - The whole numbers are also known as "the natural numbers with zero".
- **Definition:** The set of *Integers* is the set $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$.
 - The integers are also known as "the whole numbers, as well as all of their opposites"
- **Definition:** The *Opposite* of a number a, which we write as -a, is the number that is the same distance from zero as a, but on the other side of zero.
- **Example:** The numbers a = 47 and -a = -47 are opposites because they are on opposite sides of zero, and the same distance away.
- Definition: If a and b are two numbers on the number line, then the statements a < b and a > b are called Inequalities.
 - The statement a < b is read as "the number a is <u>less than</u> the number b.
 - The statement a > b is read as "the number a is *greater than* the number b.
- **Definition:** The **Absolute Value** of a number a, denoted |a|, is the distance between a and 0 on the number line.
 - Distance cannot be negative, so the absolute value of a number a is always 0 or higher.

Checking for Understanding/Closure

- True or False? $\{-27, 47\}$ is a subset of the Natural Numbers
 - False
- True or False? π is a Rational number
 - ▶ False
- True or False? $-\pi$ is the opposite of π

- ► True
- True or False? the number 8 is greater than the number -9
 - ► True
- True or False? the number 8 is the opposite the number -9
 - ▶ False
- True or False? The opposite of the opposite of a number is the number itself
 - ▶ True
- True or False? m > n if m is located to the left of n on a number line
 - ► False
- True or False? |-47| = -47
 - ▶ False

Guided Practice

• Guided Practice: Graph the number 2 on a number line.



• **Guided Practice:** Write the appropriate symbol, < or >, between the following:

> <

• **Guided Practice:** Write the appropriate symbol, < or >, between the following:

. .

• Guided Practice: What is the absolute value of 6?

▶ 6

• Guided Practice: What is the absolute value of -4?

4

• **Guided Practice:** Write the appropriate symbol, < or >, between the following:

> >

• Guided Practice: Find the missing number/numbers if possible. |?| = 0

▶ 0

• **Guided Practice:** Find the missing number/numbers if possible. |?| = -27

▶ ∅

Independent Practice

• Independent Practice: Graph the number -6 on a number line.



• Independent Practice: Write the appropriate symbol, < or >, between the following:

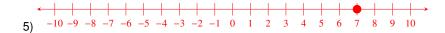
> <

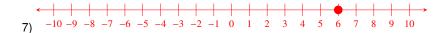
- Independent Practice: What is the opposite of 7?
 - **▶** -7
- Independent Practice: What is the opposite of -4?
 - **>** 4
- Independent Practice: Write the appropriate symbol, < or >, between the following:

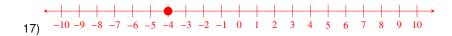
> >

- Independent Practice: Find the missing number/numbers if possible. |?| = 47
 - **►** -47, 47
- Independent Practice: Find the missing number/numbers if possible. |?| 6 = 8
 - **▶** -14, 14

Homework Assignment







23) 7 25)
$$-22$$
 27) 0 31) < 35) > 37) 15 39) 0 43) -7 45) -29 47) > 49) < 51) < 59) -5.5

61) No Number Possible (∅)

Lesson Plan

- Instructor: Luke Denton
- Subject: Elementary Algebra
- Textbook: G. Woodbury, Elementary and Intermediate Algebra, Third edition
- Section: 1.2-Operations with Integers
- Objectives: Students will be able to
 - ► Add integers
 - ▶ Subtract integers
 - ► Multiply integers
 - ► Divide integers
- Classroom Instructional Materials:
 - ► Teacher: Whiteboard and Pen; Computer and Projector; Lesson Plan.
 - ▶ Students: Pencil and Paper; Structured Notes (optional).
- Accommodations and Modifications: As documented/needed.
- Total Time Allotted: 55 minutes
 - Opening (10 minutes)
 - Focus Lesson (15 minutes)
 - Checking for Understanding and Lesson Closure (5 minutes)
 - **Guided Practice (10 minutes)**
 - (15 minutes)
- Homework Assignment: 30-45 minutes
 - Pages 13-14: 9,11,17,19,23,27,29,31,33,35,37,39,43,53,55,57,59,65,69,75,81

Opening

- The Art of Math Picture of the Day
- Class Announcements
- Homework Questions
- Introduce Lesson Objectives

Focus Lesson

Addition and Subtraction of Integers

• The Sum of two opposites

For any real number a, a + (-a) = 0.

• Adding a positive number and a negative number

Take the absolute value of each number and find the difference between these two absolute values. Note that the sign of the result is the same as the sign of the number that has the largest absolute value.

. Adding two (or more) negative numbers

Total the negative contribution of each number.

Note that the sign of the result is negative.

· Subtraction of real numbers

For any real numbers a and b, a - b = a + (-b).

Multiplication and Division of Integers

· Products of Integers

```
(Positive)-(Negative)=Negative
(Negative)-(Positive)=Negative
```

. Products of two negative integers

(Negative)-(Negative)=Positive

• Multiplication property of 0

- For any real number x, $0 \cdot x = 0$ and $x \cdot 0 = 0$.

· Quotients of Integers

- (Negative)÷(Negative)=Positive
- (Positive)÷(Positive)=Positive
- (Positive)÷(Negative)=Negative
- (Negative)÷(Positive)=Negative

· Division by Zero

Definition: Whenever an integer is divided by zero, the quotient is said to be **Undefined**.

Checking for Understanding/Closure

- True or False? The sum of two integers is always an integer
 - ▶ True
- True or False? The difference of two integers is always an integer
 - ► True

- True or False? The sum of two whole numbers is always a whole number ► True • True or False? The difference of two whole numbers is always a whole number ▶ False • True or False? 47.0=0
 - - ► True
- True or False? (-1)(-2)(-3)=6
 - ▶ False
- True or False? 4+(-7) is a whole number
 - ► False
- True or False? 47÷0=0
 - ► False

Guided Practice

- **Guided Practice:** Find the sum 12 + (-8)
 - **4**
- **Guided Practice:** Subtract 6 (-27)
 - ▶ 33
- **Guided Practice:** Multiply 5(-8)
 - **▶** -45
- Guided Practice: Multiply 0(-8)
- **Guided Practice:** Divide $(-54) \div (-6)$

Independent Practice

- **Independent Practice:** Find the sum 3 + (-11)
- **Independent Practice:** Find the sum 14 + (-6)
- **Independent Practice:** Find the sum 4 + (-17)
 - ▶ -13

```
• Independent Practice: Find the sum -2 + (-9)
```

- **▶** -11
- **Independent Practice:** Find the sum -3 + (-7)
 - ▶ -10
- Independent Practice: Subtract 11 (–7)
 - **▶** 18
- Independent Practice: Simplify 1 2 + (-3) (-4)
 - ****
- Independent Practice: Multiply 10(-6)
 - **▶** -60
- **Independent Practice:** Multiply -7(-9)
 - **▶** 63
- Independent Practice: Multiply -9(-8)
 - **▶** 72
- **Independent Practice:** Multiply -4(-10)(5)(-2)
 - -400
- Independent Practice: Divide (-33) ÷ (11)
 - **-**3
- Independent Practice: Divide (72) ÷ (-8)
 - ▶ -9

Homework Assignment

```
9) -5 11) -27 17) -11 19) 2 23) -8 27) 61 29) -9 31) -11 33) -6 35) 27 37) 1 39) $8 43) 2150 ft. 53) 0 55) 90 57) 180 59) -9 65) -14 69) undefined 75) -21 81) 0
```

Lesson Plan

- Instructor: Luke Denton
- Subject: Elementary Algebra
- Textbook: G. Woodbury, Elementary and Intermediate Algebra, Third edition
- Section: 1.3-Fractions
- Objectives: Students will be able to
 - ► Find the factor set of a natural number
 - ▶ Determine whether a natural number is prime
 - ▶ Find the prime factorization of a natural number
 - ► Simplify a fraction to lowest terms
 - ► Change an improper fraction to a mixed number
 - ► Change a mixed number to an improper fraction
- Classroom Instructional Materials:
 - ▶ **Teacher:** Whiteboard and Pen; Computer and Projector; Lesson Plan.
 - ▶ **Students:** Pencil and Paper; Structured Notes (optional).
- Accommodations and Modifications: As documented/needed.
- Total Time Allotted: 55 minutes
 - Opening (10 minutes)
 - Focus Lesson (20 minutes)
 - Checking for Understanding and Lesson Closure (5 minutes)
 - Guided Practice (5 minutes)
 - Independent Practice (15 minutes)
- Homework Assignment: 30-45 minutes
 - Page 19: 17,19,23,25,33,37,41,43,47,53,57,59,63,65,67

Opening

- The Art of Math Picture of the Day
- Class Announcements
- . Homework Questions
- Introduce Lesson Objectives

Focus Lesson

Factors

- **Definition:** To *Factor* a natural number is to express it as the product of two natural numbers.
- **Example:** 3 and 4 are the factors of 12 in the factorization $12 = 3 \cdot 4$.
- **Example:** 2 and 6 are the factors of 12 in the factorization $12 = 2 \cdot 6$.
- Definition: The collection of all factors of a natural number is called its Factor Set.
- **Example:** The factor set of 12 is the set of natural numbers {1, 2, 3, 4, 6, 12}
- Example: Write the factor set for 18

Because 18 can be factored as $1 \cdot 18$, $2 \cdot 9$, and $3 \cdot 6$, its factor set is the set of natural numbers $\{1, 2, 3, 6, 9, 18\}$.

Prime Numbers

- **Definition:** A Natural Number is **Prime** if is greater than 1 and its only two factors are 1 and itself.
- **Example:** 47 is prime because its only two factors are 1 and 47.
- **Example:** The first ten prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29.
- Definition: A natural number greater than 1 that is not prime is called a Composite Number.

The number 1 is considered to be neither prime nor composite.

• Example: Determine whether the number 26 is prime or composite.

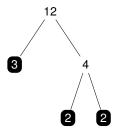
The factor set of the number 26 is the set of natural numbers $\{1, 2, 13, 26\}$. Because 26 has factors other than 1 and itself it is a composite number.

• **Example:** Determine whether the number 37 is prime or composite.

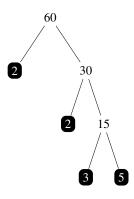
The factor set of the number 37 is the set of natural numbers $\{1, 37\}$. Because 37 has factors no other than 1 and itself, it is a prime number.

Prime Factorization

- **Definition:** When we rewrite a natural number as a product of prime factors, we obtain the **Prime Factor**ization of the number.
- **Example:** The prime factorization of 12 is $2 \cdot 2 \cdot 3$ because 2 and 3 are prime numbers and $2 \cdot 2 \cdot 3 = 12$. To find the prime factorization it is useful to use a *Factor Tree* like this one:



• **Example:** Find the prime factorization of 60



Fractions

- **Definition:** If the numerator and denominator do not have any common factors other than 1, the fraction is said to be in **Lowest Terms**.
- **Example:** The fraction 4/7 is in lowest terms because the numerator and denominator have no common factors besides 1.
- Example: The fraction 8/14 is not in lowest terms because the numerator and denominator have a common factors of 2.
- **Example:** Simplify $\frac{18}{30}$ to lowest terms.

$$\frac{18}{30} = \frac{2 \cdot 3 \cdot 3}{2 \cdot 3 \cdot 5}$$
$$= \left(\frac{2 \cdot 3}{2 \cdot 3}\right) \frac{3}{5}$$
$$= \frac{3}{5}$$

Mixed Numbers and Improper Fractions

- Definition: An Improper Fraction is a fraction whose numerator is greater than or equal to its denominator.
- Example: The fraction 7/4 is an improper fraction because the numerator is greater than the denominator.
- **Definition:** An improper fraction can be written as *Mixed Number*, which is the sum of a whole number and a proper fraction.
- Converting an Improper Fraction to a Mixed Number

To convert an improper fraction to a mixed number, begin by dividing the denominator into the numerator. The quotient is the whole number portion of the mixed number. The remainder becomes the numerator of the fractional part, while the denominator of the fractional part is the same as the denominator of the improper function.

- **Example:** The improper fraction 14/3 can be represented as the mixed number $4\frac{2}{3}$.
- Rewriting a Mixed Number as an Improper Fraction
 - 1. Multiply the whole number part by the denominator of the fractional part of the mixed number
 - 2. Add this product to the numerator of the fractional part of the mixed number
 - **3.** The sum is the numerator of the improper fraction. The denominator stays the same.

• **Example:** The mixed number $2\frac{7}{15}$ can be represented as the improper fraction 37/15.

Checking for Understanding/Closure

- True or False? The collection of all factors of a natural number is called its prime factorization.
 - ▶ False
- True or False? A number greater than 1 is prime if its only factors are 1 and itself.
 - True
- True or False? A fraction is in lowest terms if its numerator and denominator contain no common factors.
 - ► True
- True or False? An improper fraction can always be written as either a whole number or mixed number.
 - ► True
- True or False? The numerator is the number on the bottom of the fraction.
 - ► False
- True or False? 7 is a factor of 247.
 - False
- True or False? 6 is a factor of 4836.
 - ► True
- True or False? 123 is a prime number.
 - ▶ False

Guided Practice

- Guided Practice: Write the factor set for 36.
 - ► {1, 2, 3, 4, 6, 9, 12, 18, 36}
- Guided Practice: Find the prime factorization of 63.
 - **▶** 3 · 3 · 7
- Guided Practice: Simplify $\frac{4}{24}$ to lowest terms.
 - **▶** 1/6
- **Guided Practice:** Convert the improper fraction $\frac{71}{9}$ to a mixed number.
 - $ightharpoonup 7\frac{8}{9}$
- **Guided Practice:** Convert the mixed number $5\frac{4}{7}$ to an improper fraction.
 - $>\frac{39}{7}$

Independent Practice

- Independent Practice: Write the factor set for 16.
 - **▶** {1, 2, 4, 8, 16}
- Independent Practice: Write the factor set for 64.
 - **1**, 2, 4, 8, 16, 32, 64
- Independent Practice: Find the prime factorization of 90.
 - **▶** 2 · 3 · 3 · 5
- Independent Practice: List four fractions that are equivalent to $\frac{3}{4}$.
 - ▶ $\frac{6}{8}$ or $\frac{9}{12}$ or $\frac{12}{16}$ or $\frac{15}{20}$
- Independent Practice: Simplify $\frac{45}{210}$ to Lowest Terms.
- Independent Practice: Simplify $\frac{24}{384}$ to Lowest Terms.
- Independent Practice: Covert the improper fraction $\frac{121}{13}$ to a mixed number.
- Independent Practice: Convert the mixed number $8\frac{1}{6}$ to an improper fraction.

Homework Assignment

- 17) $\{1, 2, 3, 4, 6, 8, 12, 16, 24, 48\}$ 19) $\{1, 3, 9, 27\}$ 23) $\{1, 3, 9, 27, 81\}$ 25) $\{1, 31\}$ 33) $3 \cdot 13$ 37) $5 \cdot 5 \cdot 5$ 41) $3 \cdot 3 \cdot 5$ 43) Prime 47) $\frac{5}{8}$ 53) $\frac{27}{64}$ 57) $\frac{7}{13}$ 59) $\frac{19}{5}$ 63) $\frac{151}{11}$ 65) $7\frac{4}{5}$ 67) $14\frac{3}{7}$

Lesson Plan

- Instructor: Luke Denton
- Subject: Elementary Algebra
- Textbook: G. Woodbury, Elementary and Intermediate Algebra, Third edition
- Section: 1.4-Operations with Fractions
- Objectives: Students will be able to
 - ► Multiply fractions and mixed numbers
 - ► Divide fractions and mixed numbers
 - ▶ Add and subtract fractions and mixed numbers with the same denominator
 - ► Find the least common multiple (LCM) of two natural numbers
 - ▶ Add and subtract fractions and mixed numbers with different denominators
- Classroom Instructional Materials:
 - ► Teacher: Whiteboard and Pen; Computer and Projector; Lesson Plan.
 - ▶ Students: Pencil and Paper; Structured Notes (optional).
- Accommodations and Modifications: As documented/needed.
- Total Time Allotted: 55 minutes
 - Opening (10 minutes)
 - Focus Lesson (20 minutes)
 - **©** Checking for Understanding and Lesson Closure (5 minutes)
 - Guided Practice (5 minutes)
 - Independent Practice (15 minutes)
- Homework Assignment: 30-45 minutes
 - Pages 25-26: 10,11,14,19,22,25,27,30,34,37,40,42,47,50,58,62,76,80,89,93,96,97

Opening

- The Art of Math Picture of the Day
- · Class Announcements
- · Homework Questions
- Introduce Lesson Objectives

Focus Lesson

Multiplying Fractions

• To multiply fractions, after canceling any common factors, we multiply the numerators together and multiply the denominators together. To multiply mixed numbers, first convert to improper fractions.

• **Example:** Multiply $\frac{4}{11} \cdot \frac{5}{6}$

$$\frac{4}{11} \cdot \frac{5}{6} = \frac{(2 \cdot 2)}{11} \cdot \frac{5}{(3 \cdot 2)} = \frac{2}{11} \cdot \frac{5}{3} = \frac{10}{33}$$

• Example: Multiply $3\frac{1}{7} \cdot \frac{14}{55}$

$$3\frac{1}{7} \cdot \frac{14}{55} = \frac{22}{7} \cdot \frac{14}{55} = \frac{(2 \cdot 11)}{7} \cdot \frac{(2 \cdot 7)}{(5 \cdot 11)} = \frac{4}{5}$$

Dividing Fractions

- **Definition:** When we invert a fraction a/b the resulting fraction b/a is called the **Reciprocal** of the original fraction.
- **Example:** 7/4 is the reciprocal of 4/7.
- **Reciprocal Property:** For any nonzero numbers a and b, it is true that $\frac{a}{b} \cdot \frac{b}{a} = 1$
- To divide a number by a fraction, invert the divisor and multiply.
- Divide $\frac{16}{25} \div \frac{22}{15}$

$$\frac{16}{25} \div \frac{22}{15} = \frac{16}{25} \cdot \frac{15}{22} = \frac{(2 \cdot 8)}{(5 \cdot 5)} \cdot \frac{(3 \cdot 5)}{(2 \cdot 11)} = \frac{24}{55}$$

• Divide $2\frac{5}{8} \div 3\frac{3}{10}$

$$2\frac{5}{8} \div 3\frac{3}{10} = \frac{21}{8} \div \frac{33}{10} = \frac{21}{8} \cdot \frac{10}{33} = \frac{(3 \cdot 7)}{(2 \cdot 4)} \cdot \frac{(2 \cdot 5)}{(3 \cdot 11)} = \frac{35}{44}$$

Adding and Subtracting Fractions

- To add or subtract fractions that have the same denominator, we add or subtract the numerators, placing the result over the common denominator.
- **Example:** Subtract $\frac{3}{8} \frac{9}{8}$

$$\frac{3}{8} - \frac{9}{8} = \frac{-6}{8} = -\frac{3}{4}$$

• **Example:** Add $3\frac{5}{12} + 2\frac{11}{12}$

$$3\frac{5}{12} + 2\frac{11}{12} = \frac{41}{12} + \frac{35}{12} = \frac{76}{12} = \frac{19}{3} = 6\frac{1}{3}$$

Least Common Multiple

- Definition: The Least Common Multiple of two numbers is the smallest number that is a multiple of both.
- . Finding the LCM of two or more numbers
 - 1. Find the prime factorization of each number.
 - 2. Find the common factors of the numbers.
 - 3. Multiply the common factors by the remaining factors of the numbers.
- Example: Find the LCM of 24 and 30.
 - **1.** The prime factorization of $24 = 2 \cdot 2 \cdot 2 \cdot 3$ and $30 = 2 \cdot 3 \cdot 5$.
 - 2. The common factors are 2 and 3.
 - **3.** The remaining factors of the numbers are a pair of 2's from 24 and a 5 from 30. Therefore the LCM of 24 and 30 is $(2 \cdot 3) \cdot (2 \cdot 2 \cdot 5) = 120$.

Adding and Subtracting with Different Denominators

- Adding and Subtracting Fractions and Mixed Numbers with Different Denominators
 - 1. Find the LCM of the denominators.
 - Rewrite each fraction as an equivalent fraction whose denominator is the LCM of the original denominators.
 - 3. Add or subtract the numerators, placing the result over the common denominator.
 - 4. Simplify to lowest terms if possible
- **Example:** Add $\frac{5}{12} + \frac{9}{14}$
 - 1. The LCM of 12 and 14 is 84.

2.
$$\frac{5}{12} + \frac{9}{14} = \frac{5}{12} \cdot \frac{7}{7} + \frac{9}{14} \cdot \frac{6}{6}$$

$$3. \ \frac{35}{84} + \frac{54}{84} = \frac{89}{84}$$

- **4.** The fraction is already in lowest terms, or can be written as the mixed number $1\frac{5}{84}$.
- Example: $4\frac{1}{3} \frac{3}{4}$
 - 1. The LCM of 3 and 4 is 12.

2.
$$4\frac{1}{3} - \frac{3}{4} = \frac{13}{3} - \frac{3}{4} = \frac{13}{3} \cdot \frac{4}{4} - \frac{3}{4} \cdot \frac{3}{3}$$

3.
$$\frac{52}{12} - \frac{9}{12} = \frac{43}{12}$$

4. The fraction is already in lowest terms, or can be written as the mixed number $3\frac{7}{12}$.

Checking for Understanding/Closure

- **True or False?** The reciprocal property allows us to cancel factors common to both the numerator and denominator.
 - ▶ True
- True or False? When dividing with mixed numbers, we first convert to improper fractions.
 - ► True
- True or False? After adding or subtracting fractions, we always reduce the answer to lowest terms.
 - ▶ True
- **True or False?** When adding or subtracting mixed numbers, it is necessary to write the answer as a mixed number.
 - ▶ False
- True or False? The least common multiple of 24 and 30 is 720.
 - ► False
- True or False? When fractions are added or subtracted, they must first have the same numerator.
 - ▶ False

- True or False? When dividing a number by a fraction, we must invert the divisor before canceling common factors.
 - ► True
- True or False? $\frac{1}{2} + \frac{1}{2} = \frac{2}{4}$
 - ▶ False

Guided Practice

- Guided Practice: Multiply $\frac{15}{16} \cdot \frac{14}{25}$
 - $ightharpoonup \frac{21}{40}$
- Guided Practice: Divide $\frac{9}{28} \div \frac{21}{20}$
 - ► $\frac{15}{49}$
- **Guided Practice:** Subtract $\frac{7}{16} \frac{1}{16}$
 - $\rightarrow \frac{3}{8}$
- Guided Practice: Find the LCM of 12 and 20
 - ► LCM = 60
- Guided Practice: Add $\frac{4}{15} + \frac{7}{12}$
 - $rac{17}{20}$

Independent Practice

- Independent Practice: Multiply $\frac{10}{63} \cdot \frac{9}{16}$.
 - $> \frac{5}{56}$
- Independent Practice: Multiply $2\frac{2}{3} \cdot 8\frac{5}{8}$.
 - **>** 23
- Independent Practice: Divide $\frac{12}{25} \div \frac{63}{10}$.
 - $\frac{8}{105}$
- Independent Practice: Divide $\frac{20}{21} \div 2\frac{2}{3}$.
 - $\rightarrow \frac{5}{14}$

- Independent Practice: Subtract $\frac{17}{20} \frac{5}{20}$.
- Independent Practice: Add $3\frac{1}{8} + 5\frac{5}{8}$.
 - $ightharpoonup 11\frac{3}{4}$
- Independent Practice: Find the LCM of 18 and 42
- Independent Practice: Add $\frac{4}{9} + \frac{2}{15}$
- Independent Practice: Subtract $5\frac{1}{5} 3\frac{5}{6}$.
 - $ightharpoonup 1\frac{11}{30}$

Homework Assignment

10)
$$-\frac{1}{5}$$
 11) $2\frac{2}{5}$ 14) $4\frac{2}{3}$ 19) $-\frac{7}{4}$ 22) $-\frac{7}{6}$ 25) $-\frac{21}{50}$ 27) $\frac{11}{15}$ 30) $\frac{6}{5}$ 34) $-\frac{8}{21}$ 37) 84 40) 432 42) 700 47) $11\frac{1}{5}$ 50) $16\frac{7}{9}$ 58) $1\frac{1}{4}$ 62) $\frac{1}{6}$ 76) 14 80) 14 89) $1\frac{7}{12}$ cups 93) $\frac{7}{10}$ of a foot 96) $5\frac{1}{3}$ cups 97) $\frac{2}{3}$ cup

50)
$$16\frac{7}{9}$$
 58) $1\frac{1}{4}$ 62) $\frac{1}{6}$ 76) 14 80) 14 89) $1\frac{7}{12}$ cups 93) $\frac{7}{10}$ of a foot 96) $5\frac{1}{3}$ cups 97) $\frac{2}{3}$ cups

Lesson Plan

- Instructor: Luke Denton
- Subject: Elementary Algebra
- Textbook: G. Woodbury, Elementary and Intermediate Algebra, Third edition
- Section: 1.5-Decimals and Percents
- Objectives: Students will be able to
 - ▶ Perform arithmetic operations with decimals.
 - ► Rewrite a fraction as a decimal number.
 - ► Rewrite a decimal number as a fraction.
 - ► Rewrite a fraction as a percent.
 - ► Rewrite a decimal as a percent.
 - ► Rewrite a percent as a fraction.
 - ► Rewrite a percent as a decimal.
- Classroom Instructional Materials:
 - ► Teacher: Whiteboard and Pen; Computer and Projector; Lesson Plan.
 - ► Students: Pencil and Paper; Structured Notes (optional).
- Accommodations and Modifications: As documented/needed.
- Total Time Allotted: 55 minutes
 - Opening (10 minutes)
 - Focus Lesson (20 minutes)
 - Checking for Understanding and Lesson Closure (5 minutes)
 - Suided Practice (5 minutes)
 - Independent Practice (15 minutes)
- Homework Assignment: 30-45 minutes
 - Pages 31-32: 11,12,20,22,30,35,38,40,43,47,48,51,52,59,60,64,66,73,76,77

Opening

- The Art of Math Picture of the Day
- Class Announcements
- Homework Questions
- Introduce Lesson Objectives

Focus Lesson

Decimals

- Performing Arithmetic Operations With Decimals
- To add or subtract two decimal numbers, align the decimal points and add or subtract as you would integers
- **Example:** Add. 3.96 + 12.072

$$3.96 + 12.072$$
 16.032

- To multiply two decimal numbers, multiply them as you would integers. The total number of decimal places in the two factors shows how many decimal places are in the product
- **Example:** Multiply. −2.09 · 3.1

Because there are two decimal places in the first number and one in the second, we multiply as integers and then add three decimal places to the resulting product.

$$\begin{array}{c} -209 \\ \times 31 \\ \hline -6479 \end{array} \longrightarrow -6.479$$

- To divide two decimal numbers, move the decimal point in the divisor to the right so that it becomes an integer. Then move the decimal point in the dividend to the right by the same number of spaces. The decimal point in the answer will be aligned with this new location of the decimal point in the dividend.
- Example:

$$8.24 \div 0.4 \rightsquigarrow 0.4\overline{)8.24} \rightsquigarrow 4\overline{)82.4} \rightsquigarrow 4 \)82.4$$

Rewriting Fractions as Decimals and Decimals as Fractions

- Rewriting a Fraction as a Decimal Number
- To rewrite a fraction as a decimal number, we divide the numerator by the denominator.
- **Example:** Rewrite the fraction $\frac{5}{8}$ as a decimal

$$8 \ \frac{0.625}{)5.000}$$
 Therefore $\frac{5}{8} = 0.625$

• **Example:** Rewrite the fraction $\frac{23}{30}$ as a decimal

When we divide 23 by 30, the result is a decimal that does not terminate (0.7666666666...). The Pattern continues repeating the digit 6 forever. This is an example of a **repeating decimal**. We may place a bar over the repeating digit(s) to denote a repeating decimal.

$$\frac{23}{30} = 0.7\overline{6}$$

- Rewriting a Decimal Number as a Fraction
- To rewrite a terminating decimal number as fraction, identify the decimal place of the last non-zero digit and rewrite as an integer numerator over a denominator containing the appropriate power of 10. Simplify to lowest terms if possible.
- **Example:** Rewrite the decimal 0.164 as a fraction

$$0.164 = \frac{164}{1000} = \frac{41}{250}$$

Percents

- **Definition: Percents** are used to represent numbers as parts of 100. One percents, which can be written as 1%, is equivalent to
 - ▶ 1 part of 100
 - ► The fraction $\frac{1}{100}$
 - ▶ The decimal number 0.01
- Rewriting Fractions and Decimals as Percents
- To rewrite a fraction or decimal as a percent, we multiply it by 100%.
 - ! Because 100% is equal to 1, this will not change the value of the fraction or decimal number
- **Example:** Rewrite $\frac{2}{5}$ as a percent.

$$\frac{2}{5} \cdot 100\% = 2 \cdot 20\% = 40\%$$

• **Example:** Rewrite $\frac{3}{8}$ as a percent.

$$\frac{3}{8}\cdot 100\% = \frac{3}{2}\cdot 25\% = \frac{75}{2}\%$$
 (or rewritten as the mixed number $37\frac{1}{2}\%$).

• **Example:** Rewrite 0.47 as a percent.

When we multiply a decimal by 100, the result is the same as moving the decimal place to points to the right. Therefore $0.47 \cdot 100\% = 47\%$.

- Rewriting Percents as Fractions
- To rewrite a percent as a fraction, we divide it by 100 and drop the % sign. Simplify to lowest terms if possible.

Equivalently you may multiply by $\frac{1}{100}$

• Example: Rewrite 44% as a fraction.

$$44 \cdot \frac{1}{100} = \frac{11}{25}$$

• **Example:** Rewrite $16\frac{2}{3}\%$ as a fraction.

$$16\frac{2}{3} = \frac{50}{3} \rightsquigarrow \frac{50}{3} \cdot \frac{1}{100} = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$$

- Rewriting Percents as Decimals
- \bullet To rewrite a percent as a decimal, we divide it by 100 and drop the % sign.
 - ! Note that dividing a decimal number by 100 is equivalent to moving the decimal points two places **TO THE LEFT**

If a percent is greater than 100%, the equivalent decimal must be greater than 1.

• Example: Rewrite 74% as a decimal.

$$74\% \rightsquigarrow 74 \div 100 = 0.74$$

• Example: Rewrite 147% as a decimal.

$$147\% \rightsquigarrow 147 \div 100 = 1.47$$

Checking for Understanding/Closure

- True or False? The first place to the right of a decimal point is the tenths place.
 - ▶ True
- True or False? The third place to the right of a decimal point is the thousandths place.
 - ► True
- True or False? To rewrite a fraction as a decimal divide the denominator by the numerator
 - ▶ False
- True or False? To rewrite a fraction as a percent divide by 100%
 - ▶ False
- \bullet True or False? To rewrite a percent as a fraction divide it by 100% and omit the percent sign
 - True
- True or False? Percents are used to represent numbers as parts of 100.
 - ► True
- True or False? Percents, decimals and fractions are all ways to write a rational number.
 - ► True
- True or False? $1.23 \times 2 = 24.6$
 - ► False

Guided Practice

- Guided Practice: Multiply 3.12(2.7)
 - ▶ 8.424
- Guided Practice: Rewrite $\frac{5}{16}$ as a decimal
 - **0.3125**
- Guided Practice: Rewrite 0.56 as a fraction
 - $\frac{14}{25}$
- Guided Practice: Rewrite $\frac{17}{20}$ as a percent
 - 85%
- Guided Practice: Rewrite 0.26 as a percent
 - **>** 26%
- Guided Practice: Rewrite 60% as a fraction
 - ► 3/5
- Guided Practice: Rewrite 5% as a fraction

Independent Practice

- Independent Practice: Rewrite $\frac{3}{4}$ as a decimal
 - ▶ 0.75
- Independent Practice: Rewrite $\frac{5}{18}$ as a decimal
 - ▶ 0.27
- Independent Practice: Rewrite 0.425 as a fraction
 - $ightharpoonup \frac{17}{40}$
- Independent Practice: Rewrite $\frac{7}{10}$ as a percent
 - **▶** 70%
- Independent Practice: Rewrite $\frac{21}{40}$ as a percent
 - \triangleright 52 $\frac{1}{2}$ %
- Independent Practice: Rewrite 0.42 as a percent
 - **▶** 42%
- Independent Practice: Rewrite 35% as a fraction
 - $rac{7}{20}$
- Independent Practice: Rewrite $11\frac{2}{3}\%$ as a fraction
 - ► $\frac{7}{60}$
- Independent Practice: Rewrite 8% as a fraction
 - ▶ 0.08
- Independent Practice: Rewrite 240% as a fraction
 - **▶** 2.4

Homework Assignment

11) 4.7 12) -12.4 20) 302.5 22) 29.4774 30) 7.15 35)
$$-\frac{37}{50}$$
 38) $\frac{51}{250}$ 40) \$394.81 43) \$257.64 47) 80% 48) $62\frac{1}{2}\%$ 51) 675% 52) 240% 59) 320% 60) 275% 64) $\frac{1}{50}$ 66) $\frac{2}{11}$ 73) 0.07 76) 0.613 77) 4

Lesson Plan

- Instructor: Luke Denton
- Subject: Elementary Algebra
- Textbook: G. Woodbury, Elementary and Intermediate Algebra, Third edition
- Section: 1.6-Basic Statistics
- Objectives: Students will be able to
 - ► Calculate basic statistics for a set of data.
 - Construct a histogram for a set of data
- Classroom Instructional Materials:
 - ▶ **Teacher:** Whiteboard and Pen; Computer and Projector; Lesson Plan.
 - ► Students: Pencil and Paper; Structured Notes (optional).
- Accommodations and Modifications: As documented/needed.
- Total Time Allotted: 55 minutes
 - Opening (10 minutes)
 - Focus Lesson (20 minutes)
 - Checking for Understanding and Lesson Closure (5 minutes)
 - Guided Practice (5 minutes)
 - Independent Practice (15 minutes)
- Homework Assignment: 30-45 minutes
 - Pages 38-42: 10,11,13,18,20,21,25,26,29,31,32,35,40,41,47,51,53,60,67,70

Opening

- The Art of Math Picture of the Day
- Class Announcements
- Homework Questions
- Introduce Lesson Objectives

Focus Lesson

Basic Statistics

There are two basic types of statistics-those that describe the typical value for a set of data and those that describe how varied values are. The statistics that describe the typical value for a set of data are often called **measure of center**, while statistics that describe how varied a set of data is are often called **measure of spread**.

• **Definition:** The **mean** of a set of data is one measure of center for a set of data. To calculate the mean for a set of data we add all the values and then divide by how many values there are. The mean is the arithmetic average of the set of data.

$$Mean = \frac{Sum \text{ of All Values}}{Number \text{ of Values}}$$

• **Example:** Eight students were asked how far they drive to school each day. The results, in miles: 17, 8, 30, 1, 2, 5, 15, 10. Calculate the mean for this set of data

Solution:

Mean =
$$\frac{17 + 8 + 30 + 1 + 2 + 5 + 15 + 10}{8} = \frac{88}{8} = 11$$

The mean mileage for these students is 11 miles.

- **Definition:** The **median** of a set of data is another measure of center for a set of data. To find the median for a set of data, we begin by writing the values in ascending order. If a set of data has an odd number of values, the single value in the middle is the median. If a set of data has an even number of values, the median is average of the two center values
- **Example:** Find the median of the given set of values. 37, 16, 59, 18, 30, 4, 75, 46, 62.

Solution: Begin by rewriting the values in ascending order: 4, 16, 18, 30, 37, 46, 59, 62, 75. Because there is an odd number of values, the median is the single value in the center of the list. That value, and therefore the median, is 37.

• Example: Find the median of the given set of values. 65, 72, 74, 81, 71, 83, 89, 83, 53, 48, 77, 65.

Solution: Begin by rewriting the values in ascending order: 48, 53, 65, 65, 71, 72, 74, 77, 81, 82, 83, 89. Because there is an even number of values, the median is the average value of the two values in the center of the list. That value, and therefore the median, is

$$Median = \frac{72 + 74}{2} = \frac{146}{2} = 73$$

- Definition: The mode and midrange are two other measures of center for a set of data.
- The **mode** is that value that is repeated most often. If there are no repeated values, the set of data has no mode. For example, the set 1, 2, 3, 4, 5 has no mode because no values are repeated. A set of data can have more than one mode if two or more values are repeated the same number of times. For example, the set 5, 5, 7, 8, 8 has two modes-5 and 8.
- The midrange is the average of the set's minimum value and maximum value

$$Midrange = \frac{Minimum\ Value + Maximum\ Value}{2}$$

• Example: During a medical trial, the LDL cholesterol levels of 16 adult males were measured. Here are the results:

Find the mode and the midrange for this data.

Solution: Only one value, 122, has been repeated, so the mode is 122. To calculate the midrange, identify the maximum and minimum values and then average. The smallest value is 81 and the largest is 167, therefore

Midrange =
$$\frac{81 + 167}{2} = \frac{248}{2} = 124$$

Definition: The range is a measure of spread for a set of data, showing how varied the values are. To find
the range of a set of values, we subtract the minimum values in the set from the maximum value in the set.

• **Example:** Find the range for the following test scores of 9 math students.

Solution: To find the range, identify the maximum and minimum values for this set of data and then subtract the minimum value from the maximum value.

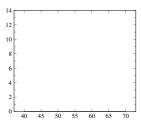
Range =
$$96 - 31 = 35$$
 points

Constructing a Histogram

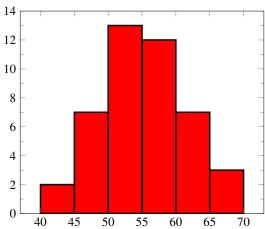
- **Definition:** A **histogram** is a graph that can be used to show how a set of data is distributed. To construct a histogram, we begin with a **frequency distribution**, which divides the data into groups, called **classes**, and lists how many times each class is represented in the set of data. To construct a histogram, begin by drawing two perpendicular axes. Mark the beginning of each class at the bottom of the graph on the horizontal axis, including the value that would be the lower limit on the next class. On the vertical axis, mark the frequencies. Make sure your axis goes at least to the highest frequency in the frequency distribution.
- Example: Construct a histogram showing the ages of U.S. Presidents at inauguration using the following frequency distribution.

| Age | Frequency |
|----------|-----------|
| 40 to 44 | 2 |
| 45 to 49 | 7 |
| 50 to 54 | 13 |
| 55 to 59 | 12 |
| 60 to 64 | 7 |
| 65 to 69 | 3 |

Solution: Begin by drawing two axes as shown. Mark the beginning of each class at the bottom of the graph (40, 45, 50, 55, 60, 65, 70), and on the vertical axis mark the frequencies (2, 4, 6, 8, 10, 12, 14).



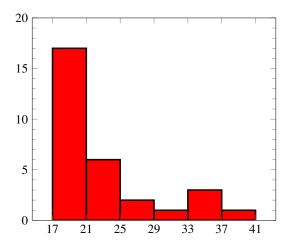
A bar is drawn above each class, and the height of the bar is determined by the frequency of that class. The first bar, from 40 to 45, should have a height of 2, the next a height of 7 and so on. Here is the histogram.



• **Example:** Draw a histogram using the following frequency distribution showing the ages of 30 students enrolled in an online algebra class

| Age | Frequency |
|----------|-----------|
| 17 to 20 | 17 |
| 21 to 24 | 6 |
| 25 to 28 | 2 |
| 29 to 32 | 1 |
| 33 to 36 | 3 |
| 37 to 40 | 1 |

Solution: On the horizontal axis, we begin the labels at 17 and increase by 4 until we reach 41. On the vertical axis, we must have labels that reach at least 17, which is the largest frequency. Finally, add the bar for each class that represents the reported frequency.



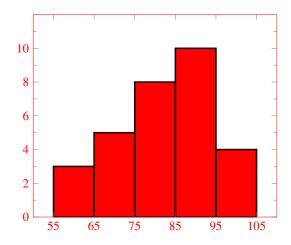
Checking for Understanding/Closure

- True or False? The mean is also known as the average
 - ▶ True
- True or False? Median and mode are both measures of center.
 - ► True
- True or False? Range and midrange are both measures of spread.
 - ▶ False
- True or False? A set of data can have more than one mode.
 - ► True
- True or False? A histogram is a graph that can be used to show how a set of data is distributed.
 - True
- True or False? The Range is always just twice the midrange.
 - ► False

Guided Practice

- Guided Practice: Calculate the mean of these six test scores: 87, 78, 90, 64, 85, 94
 - 83
- Guided Practice: Find the median for this set of eight GPA's: 3.42, 2.63, 2.95, 4.00, 3.34, 3.65, 3.12
 - ▶ 3.38
- Guided Practice: Find the mode and midrange for this set of nine ages: 21, 18, 20, 18, 19, 20, 18, 47, 19
 - ► Mode= 18, Midrange= 32.5
- **Guided Practice:** Find the range for the weights (in pounds) of 9 baseball players: 240, 430, 220, 210, 185, 220, 205, 205, 200
 - ▶ 55 pounds
- **Guided Practice:** Draw a histogram using the following frequency distribution showing the number of wins by major league teams in a year.

| Wins | Frequency |
|-----------|-----------|
| 55 to 64 | 3 |
| 65 to 74 | 5 |
| 75 to 84 | 8 |
| 85 to 94 | 10 |
| 95 to 104 | 4 |

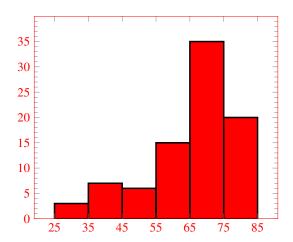


Independent Practice

- Independent Practice: Five families were asked how much they spend on groceries each month. Here are the results: \$850, \$1020, \$970,\$635, \$795. Calculate the mean for this set of data
 - **▶** \$854
- **Independent Practice:** Find the median of the given set of values: 54, 21, 39, 16, 7, 75
 - ▶ 30

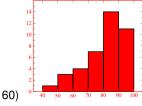
- Independent Practice: Find the median of the given set of values: 51, 3, 29, 60, 62, 25, 43, 102, 14
 - ▶ 43
- **Independent Practice:** Here are the heights, in inches, of 10 adult females: 56, 65, 66, 66, 65, 66, 65, 60, 61, 65. Find the mode and the midrange for this data.
 - ► Mode= 65, Midrange= 61
- Independent Practice: Here are the heights, in centimeters, of 8 adult males: 165, 168, 174, 179, 182, 159, 171, 180. Find the range for this data.
 - ► Range= 23 centimeters
- **Independent Practice:** Draw a histogram using the following frequency distribution showing the ages of 86 passengers on a cruise.

| Age | Frequency |
|----------|-----------|
| 25 to 34 | 3 |
| 35 to 44 | 7 |
| 45 to 54 | 6 |
| 55 to 64 | 15 |
| 65 to 74 | 35 |
| 75 to 84 | 20 |



Homework Assignment

10) 46 11) 46.75 13) 76 18) 80 20) 16,47 21) none 25) 435, 162 26) 141, 124 29) a) 52 b) 49 c) 63 d) 54.5 e) 37 31) a) 63 b) 67 c) 44,76 d) 58 e) 56 32) a) 70 b) 73 c) 86 d) 66 e) 70 35) 100.25 40) \$47,120 41) 109 47) a) \$217.80 b) \$219 c) \$199,\$219,\$259 d) \$213 e) \$172 51) a) 1003 b) 955 c) none d) 1067.5 e) 1359 53) 76



| | Score | Frequency |
|-----|----------|-----------|
| 67) | 50 to 59 | 9 |
| | 60 to 69 | 8 |
| | 70 to 79 | 11 |
| | 80 to 89 | 8 |
| | 90 to 99 | 9 |

| IQ | rrequency |
|------------|--|
| 90 to 99 | 11 |
| 100 to 109 | 13 |
| 110 to 119 | 11 |
| 120 to 129 | 6 |
| 130 to 139 | 2 |
| | 90 to 99 100 to 109 110 to 119 120 to 129 |

Lesson Plan

- Instructor: Luke Denton
- Subject: Elementary Algebra
- Textbook: G. Woodbury, Elementary and Intermediate Algebra, Third edition
- Section: 1.7-Exponents and Order of Operations
- Objectives: Students will be able to
 - ► Simplify Exponents
 - ▶ Use the order of operations to simplify arithmetic expressions
- Classroom Instructional Materials:
 - ▶ **Teacher:** Whiteboard and Pen; Computer and Projector; Lesson Plan.
 - ► Students: Pencil and Paper; Structured Notes (optional).
- Accommodations and Modifications: As documented/needed.
- Total Time Allotted: 55 minutes
 - Opening (10 minutes)
 - Focus Lesson (20 minutes)
 - Checking for Understanding and Lesson Closure (5 minutes)
 - Guided Practice (5 minutes)
 - (15 minutes)
- Homework Assignment: 30-45 minutes
 - Pages 46-47: 9,11,14,18,20,21,22,27,30,38,39,45,47,56,57,58,63,67

Opening

- The Art of Math Picture of the Day
- Class Announcements
- · Homework Questions
- Introduce Lesson Objectives

Focus Lesson

Simplifying Exponents

• **Definition:** The same number used repeatedly as a factor can be represented using what is called **Exponential Notation**. For example $7 \cdot 7 \cdot 7 \cdot 7$ can be written in exponential notation as 7^4 .

- Base and Exponent
- **Definition:** For the expression a^n the number being multiplied, in this case the real number a, is called the **Base**. The **Exponent**, in this case the natural number n, tells how many times the base is used as a factor.
- Example: Simplify 2⁷.

$$2^7 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$$
$$= 128$$

• Example: Simplify $\left(\frac{2}{3}\right)^3$.

$$\left(\frac{2}{3}\right)^3 = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3}$$
$$= \frac{8}{27}$$

! Be careful with the " - " sign. Note that $(-3)^2 = (-3)(-3) = +9$. However, $-(3)^2 = -(3 \cdot 3) = -9$.

The Order of Arithmetic Operations

In order to ensure consistent calculation of arithmetic expressions we use the order of operations agreement, which is a standard order in which arithmetic operations are to be performed, thus ensuring a single correct answer.

Order of Operations (a.k.a. "PEMDAS")

- 1. Remove Grouping Symbols- "P" Begin by simplifying all expressions within parentheses, brackets, and absolute value bars. Also perform any operations in the numerator or denominator of a fraction.
- 2. Perform Any Operations Involving Exponents- "E" After all grouping symbols have been removed from the expression, simplify any exponential expressions.
- **3. Multiply and Divide- "MD" -** These two operations have equal priority. Perform multiplications or divisions in the order that they appear from left to right.
- 4. Add and Subtract- "AS" At this point, the only remaining operations should be additions and subtractions. Again, these operations are of equal priority, and we perform them in the order they appear from left to right.
- **Example:** Simplify $(-5)^2 4(-2)(6)$.

$$(-5)^{2} - 4(-2)(6) = 25 - 4(-2)(6)$$

$$= 25 - (-48)$$

$$= 25 + 48$$

$$= 73$$

• **Example:** Simplify $8 \div 2 + 3(7 - 4 \cdot 5)$.

$$8 \div 2 + 3(7 - 4 \cdot 5) = 8 \div 2 + 3(7 - 20)$$

$$= 8 \div 2 + 3(-13)$$

$$= 4 + 3(-13)$$

$$= 4 - 39$$

$$= -35$$

- Definition: Occasionally, an expression will have one set of grouping symbols inside another set, such as
 the expression 3[7 4(9 3)] + 5 · 4. This is called *Nesting* grouping symbols. We begin by simplifying the
 innermost set of grouping symbols and work our way out from there.
- **Example:** Simplify $3[7 4(9 3)] + 5 \cdot 4$.

```
3[7-4(9-3)] + 5 \cdot 4 = 83[7-4 \cdot 6] + 5 \cdot 4
= 3[7-24] + 5 \cdot 4
= 3[-17] + 5 \cdot 4
= -51 + 20
= -31
```

Checking for Understanding/Closure

- True or False? In exponential notation, the factor being multiplied repeatedly is called the base.
 - ► True
- True or False? In exponential notation, the exponent tells us how many times the base is being multiplied.
 - ► True
- True or False? When a base is squared, it is raised to the fourth power.
 - ▶ False
- True or False? When a base is cubed, it is raised to the third power.
 - ► True
- True or False? Symbols such as () and [] are called grouping symbols
 - True
- True or False? When using the order of operations, evaluating any exponents always come before any multiplications or additions
 - ▶ True
- **True or False?** Multiplications and additions have equal priority and should be evaluated from left to right in the order that they appear.
 - ▶ False

Guided Practice

- Guided Practice: Simplify 5³
 - ▶ 125
- Guided Practice: Simplify 12 7 · 6
 - **▶** −30

- Guided Practice: Simplify $4 + 3 \cdot 5 2^6$
 - **▶** −45
- Guided Practice: Simplify $8(4^2 + 3 \cdot 5 3^2)$
 - ▶ 176

Independent Practice

- Independent Practice: Simplify 43
 - **▶** 64
- Independent Practice: Simplify $\left(\frac{1}{8}\right)^4$
 - ► $\frac{1}{4096}$
- Independent Practice: Simplify $-2 \cdot 7 + 11 3^4$
 - **►** -84
- Independent Practice: Simplify $9^2 4(-2)(-10)$
 - **1**
- Independent Practice: Simplify $20 \div 5 \cdot 10(3 \cdot 6 9)$
 - **360**
- Independent Practice: Simplify $4[3^2 + 5(2 8)]$
 - **▶** −84

Homework Assignment

9) $(-2)^4$ 11) -3^5 14) 7^2 18) 512 20) 100,000 21) 1 22) 0 27) 81 30) -128 38) 78 39) -23 45) -71 47) 25 56) $\frac{7}{30}$ 57) $-\frac{1}{4}$ 58) $\frac{6}{7}$ 63) -100 67) -1995

Lesson Plan

- Instructor: Luke Denton
- Subject: Elementary Algebra
- Textbook: G. Woodbury, Elementary and Intermediate Algebra, Third edition
- Section: 1.8-Introduction to Algebra
- Objectives: Students will be able to
 - ► Build variable expressions.
 - ► Evaluate algebraic expressions.
 - ▶ Use the commutative, associative, and distributive properties of real numbers.
 - ▶ Identify terms and their coefficients.
 - ► Simplify variable expressions.
- Classroom Instructional Materials:
 - ► Teacher: Whiteboard and Pen; Computer and Projector; Lesson Plan.
 - ▶ Students: Pencil and Paper; Structured Notes (optional).
- Accommodations and Modifications: As documented/needed.
- Total Time Allotted: 55 minutes
 - Opening (10 minutes)
 - Focus Lesson (20 minutes)
 - Checking for Understanding and Lesson Closure (5 minutes)
 - Guided Practice (5 minutes)
 - (15 minutes)
- Homework Assignment: 30-45 minutes
 - Pages 53-54: 12,14,17,18,19,22,30,31,34,37,41,43,45,47,52,56,63,67,71,76,80

Opening

- The Art of Math Picture of the Day
- · Class Announcements
- · Homework Questions
- Introduce Lesson Objectives

Focus Lesson

Variable Expressions

• **Definition:** A *Variable* is a letter or symbol that is used to represent a quantity that changes or that has an unknown value.

- **Definition:** A *Variable Expression* is a combination of one or more variables with numbers or arithmetic operations.
- Example: The following are all examples of variable expressions.

$$3a + 5$$
 $x^2 + 3x - 10$ $\frac{y+5}{y-3}$

• Example: Write an algebraic expression for "The sum of a number and 17".

Choose a variable to represent the unknown number. Let x represent the number. The expression then is x + 17. Other terms to look for that suggest addition are **plus**, **increased by**, **more than**, and **total**.

• Example: Write an algebraic expression for "Five less than a number".

Let x represent the number. The expression then is x-5. Be careful with the order of subtraction when the expression **less than** is used. "Five **less than** a number" says that we need to subtract 5 from that number. A common error is to write the subtraction in the opposite order. Other terms that suggest subtraction are **difference**, **minus**, and **decreased by**.

• Example: Write an algebraic expression for "The product of 3 and two different numbers".

Because we are building an expression involving two different unknown numbers, we need to introduce two variables. Let x and y represent the two numbers. The expression then is 3xy. Other terms that suggest multiplication are **times**, **multiplied by**, **of** and **twice**.

• Example: Four friends decide to rent a boat for the day. Assuming that all 4 friends decide to split the cost of renting the boat evenly, write a variable expression for the amount each friend will pay.

Let c represent the cost of the boat. The expression then is $c \div 4$ or $\frac{c}{4}$. Other terms that suggest division are **quotient**, **divided by**, and **ratio**.

Evaluating Variable Expressions

We often will have to evaluate variable expressions for particular values of variables. To do this, we substitute the appropriate numerical value for each variable and then simplify the resulting expression using the order of operations.

• Evaluate 2x - 7 for x = 6.

$$2x - 7$$

$$2(6) - 7 = 12 - 7$$

$$= 5$$

The expression 2x - 7 is equal to 5 for x = 6.

• **Example:** Evaluate $x^2 - 5x + 6$ for x = -5.

$$x^{2} - 5x + 6$$

$$(5)^{2} - 5(-5) + 6 = 25 - 5(-5) + 6$$

$$= 25 + 25 + 6$$

$$= 56$$

• **Example:** Evaluate $b^2 - 4ac$ for a = 3, b = -2 and c = -10.

$$b^{2} - 4ac$$

$$(-2)^{2} - 4(3)(-10) = 4 - 4(3)(-10)$$

$$= 4 - (-120)$$

$$= 4 + 120$$

$$= 124$$

Properties of Real Numbers

We now examine three important properties of real numbers.

Commutative Property

For all real numbers a and b, it is true that

$$a+b=b+a$$
 and $a\cdot b=b\cdot a$

! Note that the commutative property works with addition and multiplication but **NOT** subtraction or division.

• Associative Property

For all real numbers a, b and c, it is true that

$$(a + b) + c = a + (b + c)$$
 and $(ab)c = a(bc)$

This property states that changing the grouping of the numbers in a sum or product does not change the result. Notice that (1 + 4) + 7 is equal to 1 + (4 + 7).

$$(1+4)+7$$
 $1+(4+7)$
= 5+7 = 1+11
= 12 = 12

! Note that the associative property works with addition and multiplication but **NOT** subtraction or division.

Distributive Property

For all real numbers a, b and c, it is true that

$$a(b+c) = ab + ac$$

! Using the distributive property and factoring have the opposite effect.

This property states that we can distribute the factor outside the parentheses to each number being added in the parentheses, perform the multiplications, and then add. (This property also holds true when the operation inside the parentheses is subtraction). Consider the expression 3(5+4). The order of operations says that this is equal to $3 \cdot 9$, or 27. Here is how to simplify the expression using the distributive property.

$$3(5+4) = 3 \cdot 5 + 3 \cdot 4$$

= 15 + 12
= 27

• **Example:** Simplify 2(4 + 3x) using the distributive property.

$$2(4+3x) = 2 \cdot 4 + 2 \cdot 3x$$
$$= 8 + 6x$$

- ! Please note that the expression 8 + 6x is not equal to 14x.
- **Example:** Simplify 7(2 + 3a 4b) using the distributive property.

When there are more than two terms inside the parentheses, distribute the factor to each term. Also, it is a good idea to distribute the factor and multiply mentally.

$$(2+3a-4b) = 14+21a-28b$$

• **Example:** Simplify -4(2x - 5) using the distributive property.

When the factor outside the parentheses is negative, the negative number must be distributed to each term inside the parentheses. This will change the sign of each term inside the parentheses.

$$-4(2x-5) = (-4) \cdot 2x - (-4) \cdot 5$$
$$= -8x - (-20)$$
$$= -8x + 20$$

Simplifying Variable Expressions

- **Definition:** In an algebraic expression, a **Term** is a number, a variable, or a product of number and variables. Terms in an algebraic expression are separated by addition.
- **Example:** The expression 7x 5y + 3 has three terms: 7x, -5y, and 3
- Definition: The numerical factor of a term is its Coefficient.
- **Example:** The coefficients of the terms 7x, -5y, and 3 are 7, -5 and 3 respectively.
- **Example:** For the expression -5x + y + 3xy 19, determine the number of terms, list them, and state the coefficient for each term.

This expression has four terms: -5x, y, 3xy, and -19. These terms have coefficients of -5, 1, 3, and -19 respectively.

- **Definition:** Two terms that have the same variable factors with the same exponents, or that are both constants, are called **Like Terms**.
- **Example:** Consider the expression 9x + 8y + 6x 3y + 8z 5. There are two sets of like terms in this expression: 9x and 6x as well as 8y and -3y. There are no like terms for 8z because no other terms has z as its sole variable factor. Similarly there are no like terms for the constant term -5.
- Combining Like Terms
- When simplifying variable expressions, we can combine like terms into a single term with the same variable part by adding or subtracting the coefficients of the like term.
- **Example:** Simplify 4x + 11x by combining like terms.

These two terms are like terms because they both have the same variable factors. We can simply add the two coefficients to produce the expression 15x

$$4x + 11x = 15x$$

- A general strategy for simplifying algebraic expressions is to begin by applying the distributive property. We can then combine any like terms.
- **Example:** Simplify 8(3x 5) 4x + 7.

The first step is to use the distributive property by distributing the 8 to each term inside the parentheses. Then we will be able to combine like terms.

$$8(3x-5)-4x+7 = 24x-40-4x+7$$
$$= 20x-33$$

• **Example:** Simplify 5(9 - 7x) - 10(3x + 4).

We must make sure that we distribute the negative 10 into the second set of parentheses.

$$5(9-7x) - 10(3x+4) = 45 - 35x - 30x - 40$$
$$= -65x + 5$$

Usually we write the simplified result with variable terms preceding constant terms, but it is also correct to write 5 - 65x because of the commutative property.

Checking for Understanding/Closure

- True or False? A variable is a letter or symbol used to represent a quantity that changes or has an unknown value.
 - ▶ True
- True or False? The commutative property can be used with all four arithmetic operations.
 - ► False
- True or False? The associative property applies only to the operations of addition and multiplication.
 - True
- True or False? A term is a number, a variable, or a product of numbers and variables.
 - ► True
- **True or False?** 4 and 7x are like terms and can be added together.
 - ► False
- True or False? A coefficient is the numerical factor of a term.
 - ► True
- True or False? 2x + 4y + 7z = 13xyz
 - ▶ False

Guided Practice

- Guided Practice: Build a variable expression for the sum of a number and 21.
 - x + 21
- **Guided Practice:** Evaluate $x^2 + 5x 18$ for x = -4
 - **▶** −22
- **Guided Practice:** Simplify 4(10x)
 - ► 40*x*
- **Guided Practice:** Simplify 7(3x 8)
 - \triangleright 21*x* 56

- **Guided Practice:** For the expression $x^3 6x^2 x + 9$, determine the number of terms, identify each term, and identify the coefficient for each term.
 - Four terms
 - $x^3, -6x^2, -x, 9$
 - \triangleright 1, -6, -1, 9
- Guided Practice: Simplify 4(3x-7) + 2x + 6
 - ► 14x 22

Independent Practice

- Independent Practice: Write an algebraic expression for "9 more than a number".
 - $\triangleright x + 9$
- Independent Practice: Write an algebraic expression for "A number decreased by 25".
 - x 25
- Independent Practice: Write an algebraic expression for "Twice a number".
 - \triangleright 2x
- Independent Practice: Write an algebraic expression for "The quotient of a number and 20".
 - $\rightarrow \frac{x}{20}$
- Independent Practice: Evaluate 5x + 2 for x = 11.
 - **>** 57
- Independent Practice: Evaluate $x^2 13x 40$ for x = -8.
 - ▶ 128
- Independent Practice: Evaluate $b^2 4ac$ for a = -1, b = 5 and c = 18.
 - **9**7
- **Independent Practice:** Simplify 7(5x 4) using the distributive property.
 - \rightarrow 35x 28
- **Independent Practice:** Simplify 12(x 2y + 3z) using the distributive property.
 - ightharpoonup 12x 24y + 36z
- **Independent Practice:** Simplify -6(4x + 11) using the distributive property.
 - -24x 66
- **Independent Practice:** For the expression $x^3 x^2 + 23x 59$, determine the number of terms, list them, and state the coefficient for each term.
 - ▶ Four terms
 - $x^3, -x^2, 23x, -59$
 - **▶** 1, −1, 23, −59
- **Independent Practice:** Simplify 3x + 7y + y 5x by combining like terms.

- ► -2x + 8y
- Independent Practice: Simplify 5(2x + 3) + 9x 8.
 - ▶ 19x + 7
- Independent Practice: Simplify 3(2x-7) 8(x-9).
 - -2x + 51

Homework Assignment

- 12) x 33 14) x + 41 17) 2x + 19 18) 4x 7 19) x + y 22) $\frac{1}{2}(x + 25)$ 30) Five more than six times a number
- 31) Ten less than eight times a number 34) -9 37) 46 41) -20 43) 0 45) 49 47) 44 52) 49 56) 8x + 20 63) 16x
- 67) 7x + 471) 10x 876) 36x 2280) -x 21y + 119z + 55